

AB04: Windschiefe Geraden – Lösungen

Abstand zweier windschiefer Geraden bestimmen

Aufgabe 5:

(a)

$$G_s(1|0|-1-2s) \quad H_t(3t|5-t|1)$$

$$\vec{G_s H_t} = \begin{pmatrix} 1-3t \\ -5+t \\ -1-2s-1 \end{pmatrix} = \begin{pmatrix} 1-3t \\ -5+t \\ -2-2s \end{pmatrix}$$

$$(i) \begin{pmatrix} 1-3t \\ -5+t \\ -2-2s \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} = 0 \quad \text{und} \quad (ii) \begin{pmatrix} 1-3t \\ -5+t \\ -2-2s \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} = 0$$

↓

$$\text{I. } 4+4s=0 \rightarrow s=-1$$

$$\text{II. } 3-3t+5-t=0 \rightarrow -10t+8=0 \rightarrow t=\frac{8}{10}=\frac{4}{5}$$

$$\rightarrow G(1|0|1), H(\frac{12}{5}|\frac{21}{5}|1)$$

$$|\vec{GH}| = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{21}{5}\right)^2 + 0^2} = \sqrt{\frac{49+441}{25}} = \frac{\sqrt{490}}{5}$$

(b)

$$G_s(3+s | 1+s | 4), H_t(5 | 5-t | t)$$

$$\vec{GH_t} = \begin{pmatrix} 3+s-5 \\ 1+s-(5-t) \\ 4-t \end{pmatrix} = \begin{pmatrix} -2+s \\ -4+s+t \\ 4-t \end{pmatrix}$$

$$(1) \begin{pmatrix} -2+s \\ -4+s+t \\ 4-t \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0$$

$$(2) \begin{pmatrix} -2+s \\ -4+s+t \\ 4-t \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = 0$$

↓

$$\text{I. } -2+s-4+s+t=0$$

↓

$$\text{II. } 4-s-t+4-t=0$$

$$\text{I. } 2s+t-6=0$$

↓

$$\text{II. } -s-2t+8=0 \quad | +2\text{I}$$

$$\text{I. } 2s+t-6=0 \quad \rightarrow s=\frac{4}{3}, t=\frac{10}{3}$$

$$\text{II. } 3s-4=0$$

$$\rightarrow G(\underset{13/3}{3+\frac{4}{3}} | \underset{7/3}{1+\frac{4}{3}} | 4), H(\underset{5/3}{5 | 5-\frac{10}{3} | \frac{10}{3}})$$

$$\begin{aligned} |\vec{GH}| &= \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + 2^2} = \sqrt{\frac{8}{9} + 4} \\ &= \sqrt{\frac{8}{9} + \frac{36}{9}} \\ &= \sqrt{\frac{44}{9}} = \frac{\sqrt{44}}{3} \end{aligned}$$

Aufgabe 6:

$$G_s(3+s | 7 | 5) \quad H_t(2 | 1+t | 9)$$

$$\vec{GH_t} = \begin{pmatrix} 1+s \\ 6-t \\ -4 \end{pmatrix}$$

$$\downarrow \text{I. } 1+s = 0 \rightarrow s = -1$$

$$\text{II. } 6-t = 0 \rightarrow t = 6$$

$$\rightarrow G(-1 | 2 | 5), H(2 | 7 | 9)$$

$$|\vec{GH}| = \sqrt{4^2} = 4 < 100$$

A: Waldi hält den Abstand schon wieder nicht ein.

Aufgabe 7:

$$G_S(0|1+s|2+s), H_t(7+4t|7-5t|2t)$$

$$\overrightarrow{G_S H_t} = \begin{pmatrix} -7-4t \\ -6+s+5t \\ 2+s-2t \end{pmatrix} = \begin{pmatrix} -4t-7 \\ 5t+s-6 \\ -2t+s+2 \end{pmatrix}$$

$$\rightarrow (1) \begin{pmatrix} -4t-7 \\ 5t+s-6 \\ -2t+s+2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 0 \quad \text{und} \quad (2) \begin{pmatrix} -4t-7 \\ 5t+s-6 \\ -2t+s+2 \end{pmatrix} \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} = 0$$

↙

$$\text{I. } 5t+s-6-2t+s+2=0$$

$$\text{II. } -16t-28-25t-5s+30-4t+2s+4=0$$

↙

$$\text{I. } 3t+2s-4=0$$

$$\text{II. } -45t-3s+6=0 \quad | = 2\text{II} + 3\text{I}$$

↙

$$\text{I. } 3t+2s-4=0 \rightarrow s=2$$

$$\text{II. } -81t=0 \rightarrow t=0$$

$$\rightarrow \underline{\underline{G(0|4|5)}}, \underline{\underline{H(7|7|0)}}$$